THE EFFECT OF LATERAL SURFACE CURVATURE ON THE CHARACTERISTICS OF AXIALLY-SYMMETRIC TURBULENT BOUNDARY LAYERS

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In the calculation of axially-symmetric turbulent boundary layers it is usual to neglect the effect of lateral surface curvature on the form of the velocity profile and on other characteristics of the layer, inasmuch as the thickness δ of the boundary layer is assumed to be very much smaller than the lateral radius of curvature r_w for the axially-symmetric surface [1,2,3].

This paper propounds an approximate solution for the problem of the axially-symmetric turbulent boundary layer on a convex or concave surface, taking into account longitudinal pressure gradient and lateral surface curvature. In the limiting case $r_w \to \infty$ the solution obtained goes over into the familiar solution for the plane boundary layer on a curvilinear surface. In the limiting case of zero longitudinal pressure gradient, we obtain the solution of the problem of an axially-symmetric turbulent boundary layer on a cylinder (convex surface [4]) or in a slightly diverging duct (concave surface [5]).

An investigation of the properties of a boundary layer for zero longitudinal pressure gradient, and also of the parameters of the layer at a point of separation, allows certain conclusions to be drawn as to the effect of lateral surface curvature on the form of the velocity profile and on the properties of the turbulent boundary layer. In conclusion, calculated and experimental values for the drag coefficient of long cylinders in axial flow are compared.

On the basis of the results obtained, a solution may be built up for certain turbulent boundary-layer problems, for example, the rotational body, the axially-symmetric diffuser, the initial portion of a circular pipe, and so on.

1. The velocity profile and the resistance law. We will consider longitudinal flow of a viscous incompressible fluid about a body of revolution or in an axially-symmetric diffuser. We will choose a curvilinear system of coordinates. The x-axis is directed along a generator and the y-axis along a normal to the surface. In these coordinates the differential equations, describing the steady mean flow in a turbulent boundary layer on a cylinder or in a duct, have the form

$$ru\frac{\partial u}{\partial x} + rv\frac{\partial u}{\partial y} = -\frac{r}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial (r\tau)}{\partial y}, \qquad \frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0 \tag{1.1}$$

where r is the radius of an annular element of the boundary layer, u and v are the longitudinal and normal velocity components respectively in the boundary layer, ρ is the density of the fluid, p is the pressure, and τ is the shear stress in the boundary layer. At the same time, $r = r_w +$ cos θ for the case of a convex surface and $r = r_w - y \cos\theta$ for the case of a concave surface, where r_w is the cross-sectional radius of the convex or concave surface, and θ is the angle between the axis of symmetry and a tangent to a meridional generator of the axially-symmetric surface. Since $\cos\theta \approx 1$, we may write with a good degree of approximation

$$r = r_w \pm y \tag{1.2}$$

Here and later, the upper sign will refer to a convex and the lower to a concave surface.

To obtain a formula for the velocity distribution in the turbulent boundary layer on a convex or concave surface, we will expand the product τ in the neighborhood of the surface in a Maclaurin series,

$$r\tau = r_w\tau_w + \left[\frac{\partial (r\tau)}{\partial y}\right]_{y=0} y + \frac{1}{2}\left[\frac{\partial^2 (r\tau)}{\partial y^2}\right]_{y=0} y^2 + \ldots$$

where τ_w is the shear stress at the surface. The coefficients of the series are found by successive differentiation of the first of equations (1.1), taking into account the second equation and the boundary conditions at the surface,

$$\frac{\partial (r\tau)}{\partial y} = r_w\frac{dp}{dx}, \qquad \frac{\partial^2 (r\tau)}{\partial y^2} = 0, \qquad \frac{\partial^3 (r\tau)}{\partial y^3} = -\frac{r_w\tau_w}{\mu\nu}\frac{d\tau_w}{dx} \quad \text{for } y = 0$$

(where μ is the coefficient of viscosity and ν is the kinematic viscosity of the fluid). For not too large distances from the surface, therefore, it is correct to terms of third order to put approximately

$$r\tau = r_w\tau_w\left(1 + \frac{dp}{dx}\frac{y}{\tau_w}\right) = r_w\tau_w(1 + \lambda y^\circ) \qquad \left(y^\circ = y/\delta, \quad \lambda = \frac{dp}{dx}\frac{\delta}{\tau_w}\right) \tag{1.3}$$

Thus, the laws for the variation of shear stress in the vicinity of convex and concave surfaces are described by the same equation. Only the magnitude r entering in equation (1.3) is determined differently for convex and concave surfaces.

According to the mixing-length hypothesis the shear stress in a turbulent flow is connected with the mean-velocity gradient by the relationship

$$\tau = \rho l^2 \left(\frac{\partial u}{\partial y}\right)^2 \tag{1.4}$$

where l is the mixing length. Close to the surface, where the condition (1.3) is fulfilled, the mixing length may be taken as proportional to the distance from the surface, $l = ky$, where k is a dimensionless quantity to be determined experimentally.

Substituting (1.4), (1.2), and $l = ky$ in (1.3), we obtain the differential equation

$$\frac{\partial u}{\partial y} = \frac{v_*}{k} \frac{\sqrt{1 + \lambda y^\circ}}{y^\circ \sqrt{1 \pm \delta^\circ y^\circ}} \qquad \left(\delta^\circ = \frac{\delta}{r_w}, \quad v^* = \sqrt{\tau_w / \rho}\right)$$

where v_* is the friction velocity. Integrating this equation and determining the constant of integration from the condition that at the outer edge of the layer ($y = \delta$) the longitudinal velocity component in the boundary layer is equal to the velocity U in the outer potential stream in the case of external flow (convex surface) or to the velocity U in the core of the stream in the case of a duct (concave surface), we obtain an expression for the velocity profile

$$\frac{u}{U} = 1 + \frac{1}{z}\left\{\ln y^\circ - \ln \frac{2 + (\lambda \pm \delta^\circ) y^\circ + 2\sqrt{(1 + \lambda y^\circ)(1 \pm \delta^\circ y^\circ)}}{2 + \lambda \pm \delta^\circ + 2\sqrt{(1 + \lambda)(1 \pm \delta^\circ)}} \pm \sqrt{\frac{\lambda}{\delta^\circ}}\, F_{1,2}\right. \qquad \left(z = \frac{kU}{v_*}\right) \tag{1.5}$$

where for a convex surface

$$F_1 = \ln \frac{\lambda + \delta^\circ + 2\lambda\delta^\circ y^\circ + 2\sqrt{\lambda\delta^\circ (1 + \lambda y^\circ)(1 + \delta^\circ y^\circ)}}{\lambda + \delta^\circ + 2\lambda\delta^\circ + 2\sqrt{\lambda\delta^\circ (1 + \lambda)(1 + \delta^\circ)}}$$

and correspondingly for a concave surface

$$F_2 = \sin^{-1}\left(1 - 2\delta^\circ \frac{1 + \lambda y^\circ}{\delta^\circ + \lambda}\right) - \sin^{-1}\left(1 - 2^\circ \frac{1 + \lambda}{\delta^\circ + \lambda}\right)$$

The formulas (1.5) are suitable for arbitrary positive longitudinal pressure gradients and for negative pressure gradients which are small in absolute magnitude. Indeed, it follows from (1.3) that for $\lambda < -1$, the tangential stress becomes negative near the outer edge of the layer, with the result that formulas (1.5) lose their significance for $\lambda < -1$. Thus the range of possible values determined for the parameter λ, which characterizes the effect of longitudinal pressure gradient, is $-1 < \lambda \leqslant \infty$, and for the parameter δ^0, which characterizes the effect of lateral surface curvature, is $0 \leqslant \delta^0 \leqslant \infty$ (convex surface) and $0 \leqslant \delta^0 \leqslant 1$ (concave surface).

The formulas (1.5) only apply beyond a certain distance from the surface, inasmuch as the turbulent fluctuations die out very close to the wall and viscous friction becomes important.

To establish a resistance law connecting λ, z, δ^0 and $R_r = Ur_w/\nu$, we will consider the flow in the laminar sublayer immediately adjoining the surface. Inside this laminar sublayer the tangential stress is determined by the formula $\tau = \mu\, \partial u/\partial y$. Substituting this expression in (1.3), the resulting differential equation may be integrated and the constant of integration determined from the condition that the velocity u vanishes at the surface. We obtain

$$\frac{u}{U} = \pm \frac{k^2}{z^2} R_r \left\{ \ln(1 \pm \delta^\circ y^\circ) + \lambda \left[y^\circ \mp \frac{1}{\delta^\circ} \ln(1 \pm \delta^\circ y^\circ) \right] \right\} \tag{1.6}$$

To obtain the resistance law it is necessary to equate the velocities determined from (1.5) and (1.6) at the outer edge of the laminar sublayer ($y = \delta_l$), where these formulas are equally valid. However, it is first necessary to determine the thickness of the laminar sublayer. The thickness of the laminar sublayer may be determined from the well-known relationship of Karman, $\delta_l = a\nu/v_*$. However, in contrast to the simplest case of the turbulent boundary layer on a flat plate, for which the quantity a is constant, in the general case considered here the quantity a will depend on the longitudinal pressure gradient and on the lateral curvature of the surface.

Thus, comparing (1.5) and (1.6) for $y_l^0 = \delta_l/\delta = a/R_*$, where $R_* = v_*\delta/\nu$, we obtain the expression for the resistance law

$$z = \ln \frac{2 + (\lambda \pm \delta^\circ)\, \alpha / R_* + 2\sqrt{(1 + \lambda\alpha / R_*)(1 + \delta^\circ\alpha / R_*)}}{2 + \lambda \pm \delta^\circ + 2\sqrt{(1 + \lambda)(1 \pm \delta^\circ)}} - \ln \frac{\alpha}{R_*} \pm \tag{1.7}$$
$$\pm \frac{kR_*}{\delta^\circ} \left\{ \ln\left(1 \pm \frac{\delta^\circ\alpha}{R_*}\right) + \lambda \left[\frac{\alpha}{R_*} \mp \frac{1}{\delta^\circ} \ln\left(1 \pm \frac{\delta\alpha}{R_*}\right) \right] \right\} \mp \sqrt{\frac{\lambda}{\delta^\circ}}\, F_{1,2}\left(\lambda,\ \delta^\circ,\ \frac{\alpha}{R_*}\right)$$

in which the quantities $F_{1,2}(\lambda,\ \delta^0,\ a/R_*)$ are equal respectively to F_1 and F_2 with $y^0 = a/R_*$ and $R_* = (k/z)\delta^0 R_r$.

The expressions (1.5) for the velocity profile, together with the resistance law (1.7) and the integral relationship of Karman,

$$\frac{d\vartheta^{**}}{dx}+\frac{1}{U}\frac{dU}{dx}(2\vartheta^{**}+\vartheta^{*})=r_w\frac{c_f}{2}\qquad\left(c_f=\frac{\tau_w}{{}^1/_2\rho U^2}=\frac{2k^2}{z^2}\right) \tag{1.8}$$

allow the problem of the axially-symmetric turbulent boundary layer to be solved completely, since there are three equations to determine the three unknowns θ^*, θ^{**}, and c_f. Here

$$2\pi\vartheta^*=2\pi\int_0^\delta\left(1-\frac{u}{U}\right)\left(r_w\pm y\right)dy=2\pi r_w\delta\int_0^1\left(1-\frac{u}{U}\right)\left(1\pm\delta^\circ y^\circ\right)dy^\circ$$

$$2\pi\vartheta^{**}=2\pi r_w\delta\int_0^1\left(1-\frac{u}{U}\right)\frac{u}{U}\left(1\pm\delta^\circ y^\circ\right)dy^\circ$$

$$2\pi\vartheta=2\pi\int_0^\delta\left(r_w\pm y\right)dy=2\pi r_w\delta\left(1\pm\frac{\delta^\circ}{2}\right)$$

are respectively the displacement thickness, the momentum thickness, and the full thickness of the boundary layer. At the same time it should be noted that the parameters λ and z are not independent, since there exists between them the relationship

$$\lambda=\frac{z^2}{k^2}\delta^\circ\left(1\pm\frac{\delta^\circ}{2}\right)R_rP\qquad\left(P=-\frac{\nu}{U^2}\frac{dU}{dx}\right)$$

2. Limiting cases. We will consider a number of special cases. For $r_w\to\infty$, which corresponds to a plane curvilinear surface ($\delta^0=0,\ \lambda\neq 0$), the expressions for the velocity profile and the resistance law take the form

$$u/U=1+\frac{1}{z}\left\{\ln y^\circ+2\left(\sqrt{1+\lambda y^\circ}-\sqrt{1+\lambda}\right)-2\ln\frac{\sqrt{1+\lambda y^\circ}+1}{\sqrt{1+\lambda}+1}\right\} \tag{2.1}$$

$$z=k\alpha+\frac{k\alpha^2}{2}\frac{\lambda}{R_*}+2\left(\sqrt{1+\lambda}-\sqrt{1+\alpha\lambda/R_*}\right)-\ln\frac{\alpha}{R_*}+2\ln\frac{\sqrt{1+\alpha\lambda/R_*}+1}{\sqrt{1+\lambda}+1}$$

For $\lambda=0$ the expressions (1.5) and (1.7) are transformed into the velocity profile and the resistance law for the axially-symmetric turbulent boundary layer on a cylinder [4] and in a slightly diverging duct [5].

$$u/U=1+\frac{1}{z}\left\{\ln y^0-2\ln\frac{\sqrt{1\pm\delta^0 y}+1}{\sqrt{1\pm\delta^0}+1}\right\}$$

$$z=2\ln\frac{\sqrt{1\pm\delta^0\alpha/R_*}+1}{\sqrt{1\pm\delta^0}+1}-\ln\frac{\alpha}{R_*}\pm\frac{kR_*}{\delta^0}\ln(1\pm\alpha\delta^0/R_*) \tag{2.2}$$

Finally, for $\lambda\to 0$ and $\delta^0\to 0$ we obtain the well-known expressions for the velocity profile and resistance law of a flat plate.

$$u/U = 1 + \frac{1}{z}\ln y^0, \qquad \delta = C\frac{\nu}{U}ze^z \qquad \left(C = \frac{a}{k}e^{-ka}\right) \tag{2.3}$$

At a separation point of an axially-symmetric turbulent boundary layer the parameters λ and z become infinite, and formulas (1.5) and (1.7) become indeterminate. On resolving the indeterminacy in question, we obtain formulas for the velocity profile and resistance law at a separation point of an axially-symmetric boundary layer on a convex surface

$$\frac{u}{U} = \frac{\ln[1 + 2\delta^0 y^0 + 2\sqrt{\delta^0 y^0(1+\delta^0 y^0)}]}{\ln[1 + 2\delta^0 + 2\sqrt{\delta^0(1+\delta^0)}]}, \qquad \sqrt{\frac{dp}{dx}\frac{\delta}{\rho U^2}} = \frac{k\sqrt{\delta^0}}{\ln[1 + 2\delta^0 + 2\sqrt{\delta^0(1+\delta^0)}]} \tag{2.4}$$

and similarly, on a concave surface

$$\frac{u}{U} = \frac{\pi - 2\sin^{-1}(1 - 2\delta^0 y^0)}{\pi - 2\sin^{-1}(1 - 2\delta^0)}, \qquad \sqrt{\frac{dp}{dx}\frac{\delta}{\rho U^2}} = \frac{k\sqrt{\delta^0}}{\pi/2 - \sin^{-1}(1-\delta^0)} \tag{2.5}$$

For $\delta \to 0$ formulas (2.4) and (2.5) become the velocity profile and resistance law at a separation point for a plane turbulent boundary layer:

$$\frac{u}{U} = (y^0)^{1/2}, \qquad \sqrt{\frac{dp}{dx}\frac{\delta}{\rho U^2}} = \frac{k}{2} \tag{2.6}$$

In all this it is remarkable that the experimentally determined coefficient k does not enter into the expressions (2.4)-(2.6) for the velocity profile at a point of separation in the boundary layer. The second formula for each pair (2.4)-(2.6), like the dimensionless ratios $H^* = \theta^*/\theta$, $H^{**} = \theta^{**}/\theta$ and $H = \theta^*/\theta^{**}$ computed using the first formula, may be considered as a condition for separation of the turbulent boundary layer on a convex, concave, or plane surface.

3. The effect of lateral surface curvature on the form of the velocity profile, on the frictional resistance and on the separation parameters for a turbulent boundary layer. Certain conclusions may be drawn about the effect of lateral curvature of a convex or concave surface on the properties of the turbulent boundary layer by considering the case of zero longitudinal pressure gradient, corresponding to flow along a cylinder or in a slightly diverging duct (diffuser with zero longitudinal pressure gradient). In the latter case, since the thickness of the boundary layer on the wall of the duct grows more rapidly along the channel than the radius, the boundary layers will merge with each other at some distance from the channel entrance such that $\delta = r_w$.

For the case under consideration,

$$\frac{\vartheta^*}{r_w^2} = \frac{1}{z}\left[\pm \frac{1 \pm Ae^z}{1 \mp Ae^z} \pm \frac{1}{3}\left(\frac{1 \pm Ae^z}{1 \mp Ae^z}\right)^3 \mp \frac{4}{3}\right]$$

$$\frac{\vartheta^{**}}{r_w^2} = \frac{\vartheta^*}{r_w^2} \mp \frac{8}{3z^2}\left[\frac{Ae^z}{(1 \mp Ae^z)^2} - 2\ln(1 \mp Ae^z)\right] \qquad (3.1)$$

$$\frac{\delta}{w} = \frac{4Ae^z}{(1 \mp Ae^z)^2}, \qquad A = \frac{\pm\sqrt{1 \pm \alpha/\eta_w} \mp 1}{\sqrt{1 \pm \alpha/\eta_w} + 1}\left(1 \mp \frac{\alpha}{\eta_w}\right)^{\mp k\eta_w}, \qquad \eta_w = \frac{v_* r_w}{\nu} = \frac{k}{z}R_r$$

For $r_w \to \infty$ we will have

$$\lim_{r_w \to \infty}\frac{\vartheta^*}{\vartheta} = \frac{\delta^*}{\delta} = \int_0^1\left(1 - \frac{u}{U}\right)dy^0 = \frac{1}{z}, \qquad \lim_{r_w \to \infty}\frac{\vartheta^{**}}{\vartheta} = \frac{\delta^{**}}{\delta} = \int_0^1 \frac{u}{U}\left(1 - \frac{u}{U}\right)dy^0 = \frac{1}{z} - \frac{2}{z^2} \qquad (3.2)$$

where δ^* and δ^{**} are the displacement and momentum thickness respectively for the plane boundary layer.

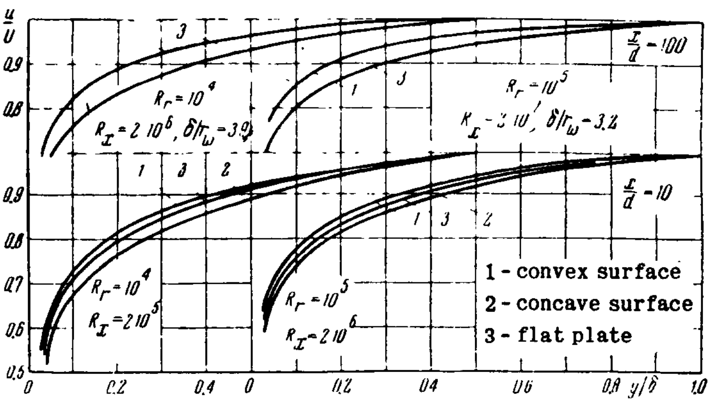

Fig. 1.

In the case of zero longitudinal pressure gradient, $dp/dx = -\rho U dU/dx = 0$, the momentum-integral relationship (1.8) is simplified and for a convex surface (cylinder) gives

$$\frac{x}{r_w} = \frac{1}{k^2}\int_0^\chi z^2 d\chi \qquad \left(\chi = \frac{\vartheta^{**}}{r_w^2}\right) \qquad (3.3)$$

and for a concave surface (duct)

$$\frac{x}{r_0} = \frac{1}{k^2} \int_0^{\chi_0} \frac{z^2}{r_w{}^0} d\chi_0 \qquad \left(\chi_0 = \frac{\vartheta^{**}}{r_0{}^2}\right) \tag{3.4}$$

where r_0 is the channel radius at the entrance section and $r_w{}^0 = r_w r_0$. The formulas (3.3) and (3.4), together with the first two of the formulas (3.1), allow the problem to be formulated completely for the case of external flow, as the number of unknowns is equal to the number of equations. For the case of the duct the number of unknowns is greater, since besides $\theta^{**}/r_0{}^2$ and z there is also involved the quantity $r_w{}^0$, for whose determination it is necessary to use the equation of efflux

$$r_w{}^0 = (1 - 2\vartheta^* / r_w{}^2)^{-1/2}$$

The initial value $z = z_0$ in formulas (3.3) and (3.4) is determined by the condition that the momentum thickness is equal to zero at $x = 0$. Because then $\delta/r_w \to 0$, the corresponding magnitude z_0 is equal to two, just as in the case of the plate (cf. the second formula of (3.2)). For $r_w \to \infty$ formulas (3.3) and (3.4) become the well-known relationship defining the function $z = z(R_x)$ for a plate with a fully turbulent boundary layer,

$$R_x = \frac{Ux}{\nu} = C_1(z^2 - 4z + 6)\, e^z - C_2 \qquad \left(C_1 = \frac{\alpha}{k^3} e^{-k\alpha},\ C_2 = 2C_1 e^2\right) \tag{3.5}$$

During the integration of (3.3) and (3.4) the quantities k and α were taken as identical with the corresponding constants for a plane turbulent boundary layer, $k = 0.392$ and $\alpha = 11.5$. The calculations were carried out for values of the number $R_r = 10^4$, 10^5 and 10^6.

The curves of u/U against y/δ plotted in Fig. 1 for $x/2r_w = 10$ and $R_r = 10^4$ and 10^5 show the nature of the effect of lateral surface curvature on the form of the velocity profile in a turbulent boundary layer. For comparison, the same figure also shows the velocity profile in a turbulent boundary layer on a flat plate for the same value of R_x as for the convex or concave surface; $R_x = Ux/\nu = R_r(x/r_w) = 2R_r(x/d)$.

It is seen that the velocity profile in the axially-symmetric turbulent boundary layer on a convex surface is more full, and on a concave surface is less full, than the velocity profile in the turbulent boundary layer on a flat plate. This difference in the velocity profile is particularly noticeable at values of x/d sufficiently large so that the thickness of the boundary layer becomes comparable to the radius r_w (concave surface) or appreciably exceeds it (convex surface) (Fig. 1).

From Fig. 1, and also from equations (3.1), (3.3), and (3.4), it follows that the form of the velocity profile in an axially-symmetric turbulent boundary layer is determined not only by the Reynolds number R_x calculated for the length x appropriate for a flat plate, but also by a second parameter R_r or by the combination $R_x/R_r = x/r_w = 2x/d$.

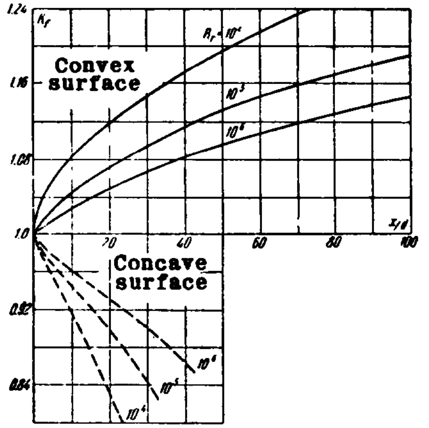

Fig. 2.

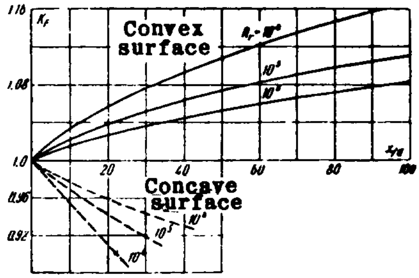

Fig. 3.

The change in the form of the velocity profile for an axially-symmetric boundary layer, taken together with the altered geometry of the axially-symmetric flow, leads to the values of the local and friction coefficients c_f and c_F for convex and concave surfaces which differ from the corresponding coefficients (c_{fpl} and c_{Fpl}) for a flat plate at the same value of R_x. The characteristic variation of the parameters $k_f = c_f/c_{fpl}$ and $k_F = c_F/c_{Fpl}$ with x/d is presented in Figs. 2 and 3 for three values of $R_r = 10^4$, 10^5 and 10^6. The data cited indicate that under certain conditions the effect of lateral surface curvature on the friction drag may prove to be very large.

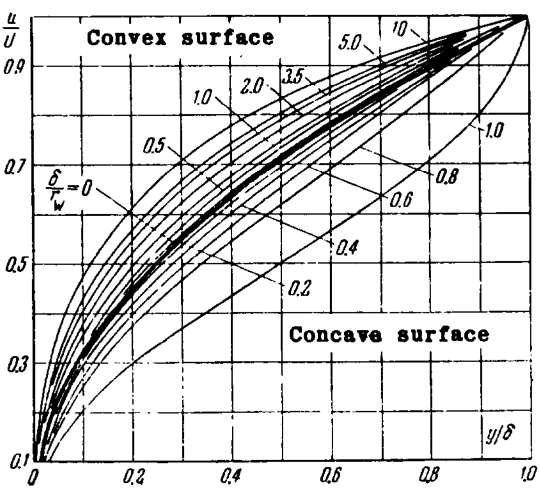

Fig. 4.

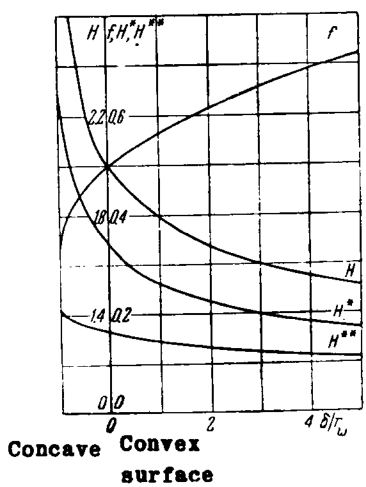

Fig. 5.

An investigation of the effect of lateral surface curvature on the properties of the layer at a point of separation is also of interest.

Figure 4 shows velocity profile computed according to formulas (2.4)-(2.6) for a separation point in a turbulent layer, and Fig. 5 shows the corresponding dependence of H^*, H^{**}, H and $f = 1/k \sqrt{(dp/dx)(\delta/\rho U^2)}$ on δ/r_w.

It follows that at a separation point in an axially-symmetric turbulent boundary layer lateral curvature makes the velocity profile more full on a convex surface, but less full on a concave surface, compared to the corresponding profile in a plane turbulent layer. It should be observed that the magnitudes $H^* = 1/3$, $H^{**} = 1/6$ and $H = 2$ at a separation point in a plane turbulent layer are sufficiently close to the corresponding experimental values.

The expressions (1.5) for the velocity profile and (1.7) for the resistance law in an axially-symmetric turbulent boundary layer, together with the quantitative results presented above, point to the existence of a certain analogy between the effects of lateral surface curvature and of longitudinal pressure gradient. In particular, lateral curvature of a convex surface alters the form of the velocity profile and the frictional drag coefficients in the same direction as a negative longitudinal pressure gradient (effuser effect). On the other hand, lateral curvature of a concave surface has an effect on the velocity profile and friction drag analogous to the effect of a positive longitudinal pressure gradient (diffuser effect).

H^*, H^{**}, H и $f = \frac{1}{k}\sqrt{\frac{dp}{dx}\frac{\delta}{\rho U^2}}$ от δ/r_w

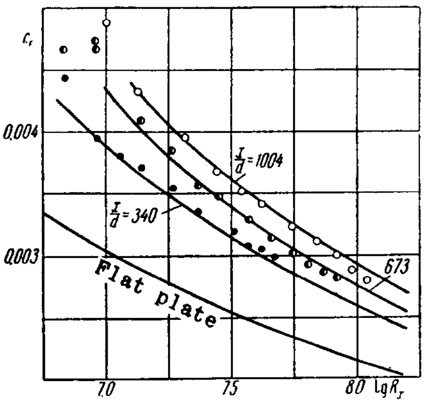

Fig. 6.

4. Comparison of theory and experiment. In conclusion, we will compare experimental values of the mean friction coefficient for long cylinders [6] with the corresponding theoretical coefficients computed according to the method set forth above (Fig. 6). As is seen from Fig. 6, a sufficiently good agreement between the experimental and computed values of the friction coefficient for the cylinders was found.

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